Control

Lec 3, Report 2

sec 3

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Q: Given:

$$GH(s) = \frac{K(s+4)(s+6)}{s(s+2)}$$

① Draw the root locus.

② Find the rang of K for Stability

③ Find K at $t_s = 2$ sec $= \frac{4}{\zeta \omega_n}$

④ Find K at $\zeta = \frac{\sqrt{3}}{2} \Rightarrow \phi = \cos^{-1} \quad \varphi = 30°$

Solution

① Poles $\Rightarrow$ 0, 2

Zeros $\Rightarrow$ -4, -6

② S-plane

③ Real Parts:

[1] $0 \longrightarrow -2$

[2] $-4 \longrightarrow -6$

④ Asymptotec lines $\Rightarrow$ useless $n_z = n_p$

⑤ Breaking Points

Ch. eq $\Rightarrow 1 + GH(s) = 0$

$$1 + \frac{K(s+4)(s+6)}{s(s+2)} = 0 \quad ①$$

rewrite ① as

$$K = -\frac{s(s+2)}{(s+4)(s+6)}$$

$$\frac{dK}{ds} = 0$$

$$\frac{dK}{ds} = -\frac{(s^2+10s+24)(2s) - (s^2+2s)(2s+10)}{(s^2+10s+24)^2} = 0$$

$$(s^2+10s+24)(2s) - (s^2+2s)(2s+10) = 0$$

$$8s^2 + 48s + 48 = 0$$

$$s_{1,2} = -1.267 \text{ (Breakout)}, \; -4.732 \text{ (Breakin)}$$

⑥ System is stable for All $K > 0$

⑦ Center of circle $= -\left(\frac{4.732 - 1.267}{2} + 1.27\right)$

$\simeq -3$

$$r = \frac{4.732 - 1.267}{2} = 1.732 \simeq 1.75$$

⑨ For $t_s = 2s$

$$K_1 = \frac{4.9 * 2.85}{4.9 * 8.5} = 0.335$$

$t_s = \frac{4}{\zeta \omega_n} = 2$

$\Rightarrow \zeta \omega_n = 2$

⑩ For $\zeta = \frac{\sqrt{3}}{2}$ (there exist two values)

$$K_2 = \frac{3.5 * 2}{5.35 * 9.25} = 0.141$$

$\phi = \cos^{-1} \zeta = 30$

$$K_3 = \frac{7 * 4}{7 * 4} = 1$$

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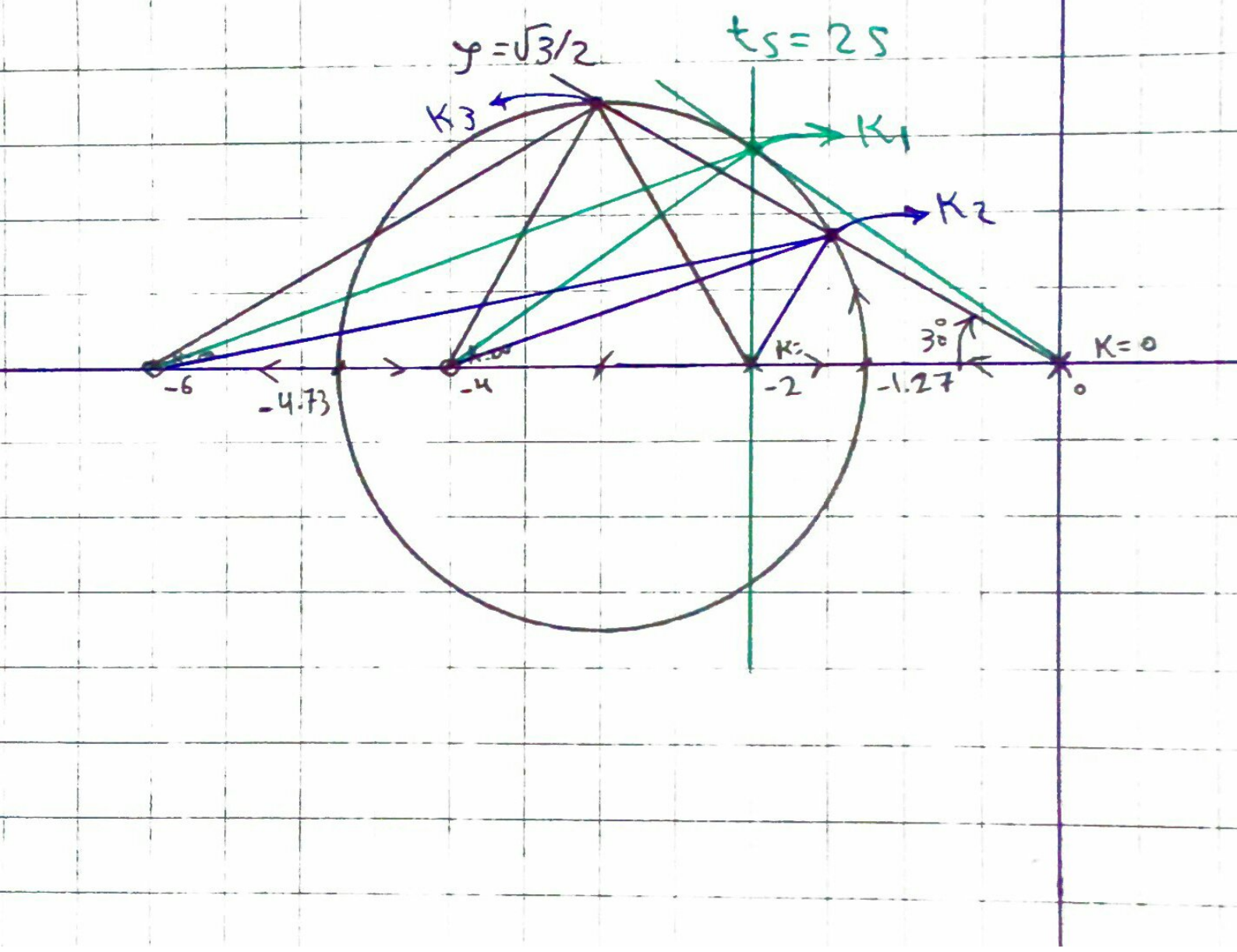
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